AP Calculus AB **Name:** *Answer Key*

Unit 2 – Day 2 – Assignment

1. The line defined by the equation $2y+3=-\frac{2}{3}(x-3)$ is tangent to the graph of $g(x)$ at $x=-3$.

derivative

What is the value of $\boxed{\lim_{x\to -3}\frac{g(x)-g(-3)}{x+3}}$? Show your work and explain your reasoning.

need slope:

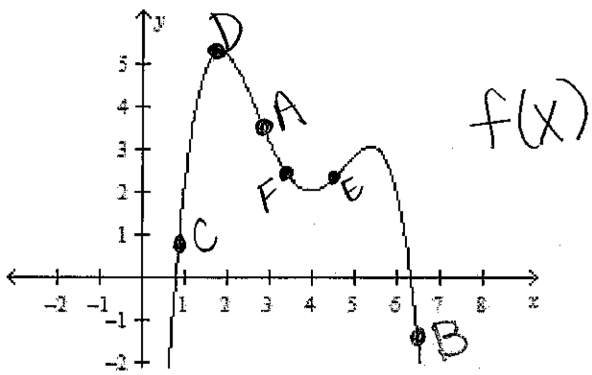
$2y+3=-\frac{2}{3}(x-3)$

$2y+3=-\frac{2}{3}x+2$

$2y=-\frac{2}{3}x-1$ → $y=-\frac{1}{3}x-\frac{1}{2}$ slope $=\boxed{-1/3}$

slope of tangent line at $x=-3$

Use the graph of $f(x)$ pictured to the right to perform the actions in exercises 2 – 6. Give written explanations for your choices.



2. Label a point, A, on the graph of $y=f(x)$ where the derivative is negative. $f'(x)<0$ when $f(x)$ is decreasing

3. Label a point, B, on the graph of $y=f(x)$ where the value of the function is negative.

f(x) is negative when graph of f(x) is below the x-axis.

4. Label a point, C, on the graph of $y=f(x)$ where the derivative is greatest in value.

$f'(x)>0$ & greatest slope = steepest line

5. Label a point, D, on the graph of $y=f(x)$ where the derivative is zero.

$f'(x)=0$ at rel. max or rel. min

6. Label two different points, E and F, on the graph of $y=f(x)$ where the values of the derivative are opposites.

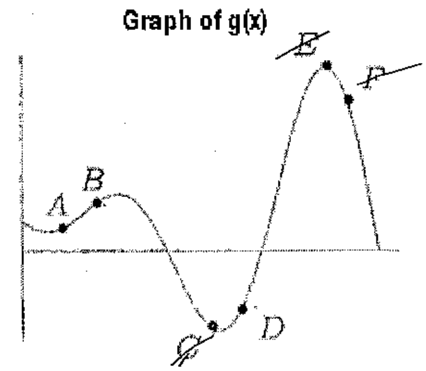
E → function increasing

F → function decreasing

opp. values of slope

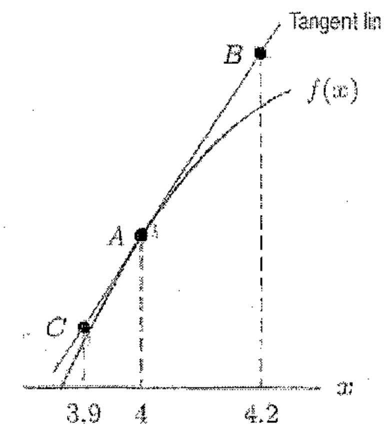
7. Match the points on the graph of $g(x)$ with the value of $g'(x)$ in the table.

	Value of $g'(x)$	Point on $g(x)$
decr.	-3	F $\leftarrow$ steeper
decr.	-1	C
max/min	0	E
incr	$\frac{1}{2}$	A
incr	1	B
incr	2	D



g(x)

8. The function to the right is such that $h(4) = 25$ and $h'(4) = 1.5$. Find the coordinates of A, B, and C.



$(4,25) \rightarrow$ P.O.T $\quad 1.5 \leftarrow$ S.O.T

$y - 25 = 1.5(x-4)$

$\boxed{A = (4,25)}$

$\boxed{B = (4.2, 25.3)} \rightarrow y - 25 = 1.5(4.2-4)$
$y = 25.3$

$\boxed{C = (3.9, 24.85)} \rightarrow y - 25 = 1.5(3.9-4) \quad y = 24.85$

For exercises 9 – 11, use the function $f(x) = \dfrac{1}{x+1}$.

9. Find $f'(x)$.

$$\lim_{h \to 0} \frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h}$$

$$\frac{1}{x+h+1}\frac{(x+1)}{(x+1)} + \frac{-1}{x+1}\frac{(x+h+1)}{(x+h+1)}$$

$$= \frac{\cancel{x}+\cancel{1}-\cancel{x}-h-\cancel{1}}{(x+1)(x+h+1)} = \frac{\frac{-h}{(x+1)(x+h+1)}}{h}$$

$$\frac{-\cancel{h}}{(x+1)(x+h+1)} \cdot \frac{1}{\cancel{h}}$$

$$\lim_{h \to 0} \frac{-1}{(x+1)(x+h+1)} = \frac{-1}{(x+1)(x+0+1)}$$

$$\boxed{f'(x) = \frac{-1}{(x+1)^2}}$$

10. Find the equation of the tangent line drawn to the graph of $f(x)$ at $x = 0$.

P.O.T $\rightarrow f(0) = \frac{1}{0+1} = 1 \quad (0,1)$

S.O.T $\rightarrow f'(0) = \frac{-1}{(0+1)^2} = -1$

$\boxed{y - 1 = -1(x+0)}$

11. Find the equation of the normal line drawn to the graph of $f(x)$ at $x = 0$.

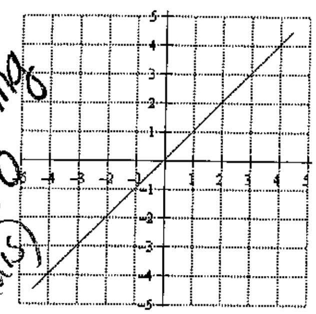
Slope of normal $= +1$

$(0,1)$

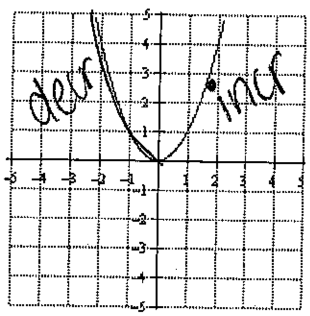
$\boxed{y - 1 = 1(x-0)}$

12. Given below are graphs of four functions—$f(x)$, $g(x)$, $h(x)$, and $p(x)$. Below those graphs are graphs of their derivatives. Label the graphs below as $f'(x)$, $g'(x)$, $h'(x)$, and $p'(x)$.

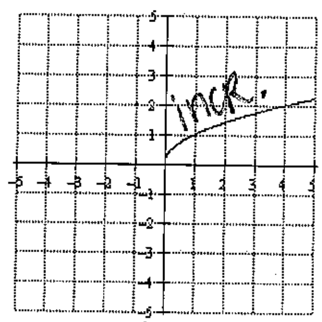
increasing
$f'(x) > 0$
(above x-axis)



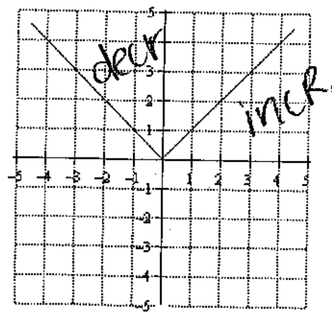
f(x)



g(x)

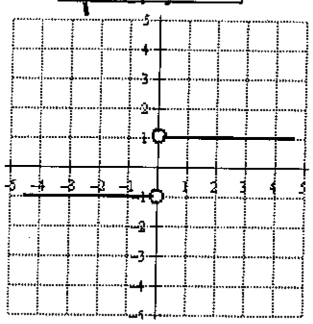


h(x)

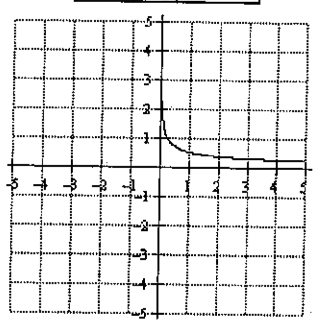


p(x)

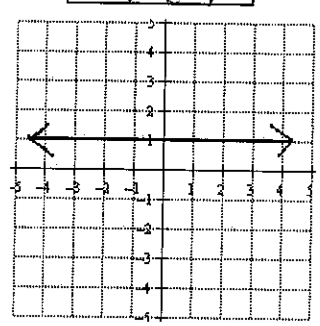
$\boxed{p'(x)}$



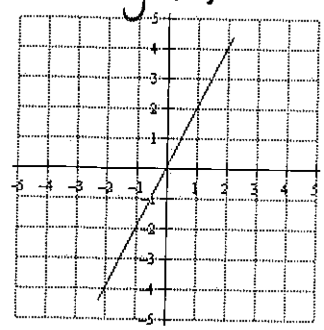
$\boxed{h'(x)}$



$\boxed{f'(x)}$



$\boxed{g'(x)}$



The table below represents values on the graph of a cubic polynomial function, $h(x)$. Use the table to complete exercises 13 – 15.

x	–3	–2	–1	0	1	2	4
$h(x)$	–24	0	8	6	0	–4	18

13. Two of the zeros of $h(x)$ are listed in the table. Between which two values of x does the Intermediate Value Theorem guarantee that a third value of x exists such that $h(x) = 0$? Explain your reasoning.

h(x) must have a zero (y=0) b/c x=2 & x=4 b/c h(2) = -4 & h(4) = 18 so h(x)=0 is b/t h(2) & h(4).

14. Estimate the value of $h'(1.5)$. Based on this describe the behavior of $h(x)$ at $x = 1.5$. Justify your reasoning.

(1,0) & (2,-4)

$$h'(1.5) \approx \frac{-4-0}{2-1} = \frac{-4}{1} \approx -4$$

h(x) is decreasing at x=1.5

15. Estimate the value of $h'(-1.75)$. Based on this value, describe the end Behavior of $h(x)$ at $x = -1.75$. Justify your reasoning. (-2,0) & (-1,8)

$$h'(-1.75) \approx \frac{8-0}{-1-(-2)} = \frac{8}{1} = 8$$

h(x) is increasing at x = -1.75