AP Calculus AB
Unit 2 – Day 1 – Assignment

Name: *Answer Key*

Find $\lim\limits_{h \to 0} \frac{f(x+h)-f(x)}{h}$ for each of the functions below. Then, find the equation of the tangent line to the graph of $f(x)$ at the given value of x.

1. $f(x) = x^3 + 2x$

$$\lim_{h \to 0} \frac{(x+h)^3 + 2(x+h) - (x^3+2x)}{h}$$

$$(x+h)(x^2+2xh+h^2)$$

$$x^3 + 2x^2h + xh^2 + x^2h + 2xh^2 + h^3$$

$$\cancel{x^3} + 3x^2h + 3xh^2 + h^3) + \cancel{2x} + 2h - \cancel{x^3} - \cancel{2x}$$

$$3x^2h + 3xh^2 + h^3 + 2h$$

$$\frac{\cancel{h}(3x^2 + 3xh + h^2 + 2)}{\cancel{h}}$$

$$\lim_{h \to 0} (3x^2 + 3xh + h^2 + 2)$$

$$= 3x^2 + 3x(0) + (0)^2 + 2$$

$$\boxed{f'(x) = 3x^2 + 2}$$

2. Find the equation of the line tangent to the graph of $f(x) = x^3 + 2x$ at $x = -1$.

• P.O.T $\rightarrow f(-1) = (-1)^3 + 2(-1)$
$= -1 - 2 = -3$
$(-1, -3)$

• S.O.T $\rightarrow f'(-1) = 3(-1)^2 + 2 = 5$

$$\boxed{y + 3 = 5(x+1)}$$

3. $f(x) = \sqrt{3-x}$

$$\lim_{h \to 0} \frac{\sqrt{3-(x+h)} - \sqrt{3-x}}{h}$$

$$\frac{(\sqrt{3-x-h} - \sqrt{3-x})}{(h)} \cdot \frac{(\sqrt{3-x-h} + \sqrt{3-x})}{(\sqrt{3-x-h} + \sqrt{3-x})}$$

$$\frac{3-x-h + \cancel{(3+x)}}{(h)(\sqrt{3-x-h} + \sqrt{3-x})} = \frac{-\cancel{h}}{(\cancel{h})(\sqrt{3-x-h} + \sqrt{3-x})}$$

$$\lim_{h \to 0} \frac{-1}{\sqrt{3-x-h} + \sqrt{3-x}} = \frac{-1}{\sqrt{3-x-0} + \sqrt{3-x}}$$

$$\boxed{f'(x) = \frac{-1}{2\sqrt{3-x}}}$$

4. Find the equation of the line tangent to the graph of $f(x) = \sqrt{3-x}$ at $x = -6$.

• P.O.T $\rightarrow f(-6) = \sqrt{3--6} = \sqrt{9} = 3$
$(-6, 3)$

• S.O.T $\rightarrow f'(-6) = \frac{-1}{2\sqrt{3--6}} = \frac{-1}{2\sqrt{9}} = -\frac{1}{6}$

$$\boxed{y - 3 = -\frac{1}{6}(x+6)}$$

For problems 5 – 9, use the function $f(x) = \dfrac{x}{x+2}$.

5. Find $f'(x)$ by finding $\lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$.

$$\lim_{h\to 0} \frac{\frac{x+h}{x+h+2} - \frac{x}{x+2}}{h}$$

$$\frac{(x+h)(x+2)}{(x+h+2)(x+2)} + \frac{-x(x+h+2)}{(x+2)(x+h+2)}$$

$$= \frac{\cancel{x^2}+\cancel{2x}+\cancel{xh}+2h-\cancel{x^2}-\cancel{xh}-\cancel{2x}}{(x+2)(x+h+2)}$$

$$= \frac{2\cancel{h}}{(x+2)(x+h+2)} \cdot \frac{1}{\cancel{h}}$$

$$= \frac{2}{(x+2)(x+h+2)}$$

$$\lim_{h\to 0} \frac{2}{(x+2)(x+h+2)} = \frac{2}{(x+2)(x+0+2)}$$

$$\boxed{f'(x) = \frac{2}{(x+2)^2}}$$

6. Find the slope of the tangent line drawn to the graph of $f(x)$ at $x = -2$.

$$f'(-2) = \frac{2}{(-2+2)^2} = \frac{2}{0}$$

$\boxed{\text{undefined}}$

7. Find the slope of the tangent line drawn to the graph of $f(x)$ at $x = -1$.

$$f'(-1) = \frac{2}{(-1+2)^2} = \frac{2}{1} = \boxed{2}$$

8. Find the equation of the tangent line drawn to the graph of $f(x)$ at $x = -1$.

slope = 2

point → $f(-1) = \dfrac{-1}{-1+2} = \dfrac{-1}{1} = -1$

$(-1, -1)$

$\boxed{y+1 = 2(x+1)}$

9. Find $\lim_{x\to a} \dfrac{f(x)-f(a)}{x-a}$, where $a = -1$.

$$\lim_{x\to -1} \frac{\frac{x}{x+2} - \left(\frac{-1}{-1+2}\right)}{x-(-1)}$$

$$\frac{x}{x+2} + 1 \rightarrow \frac{x}{x+2} + \frac{x+2}{x+2} = \frac{2x+2}{x+2}$$

$$\frac{2(x+1)}{x+2}$$

$$\lim_{x\to -1} \frac{2\cancel{(x+1)}}{x+2} \cdot \frac{1}{\cancel{x+1}} \rightarrow \lim_{x\to -1} \frac{2}{x+2} = \frac{2}{-1+2} = \frac{2}{1}$$

$$= \boxed{2}$$ ✓