AP Calculus AB Exam - 2008

Section 1 : No calculator (#'s 1-28)

AP Calculus 2008 Multiple Choice

1. $\lim_{x\to\infty}\frac{(2x-1)(3-x)}{(x-1)(x+3)}$ is

(A) -3 $\boxed{\text{(B) } -2}$ (C) 2 (D) 3 (E) nonexistent

Divide by highest exponent in denominator.

$\lim_{x\to\infty} \frac{-2x^2+7x-3}{x^2+2x-3}$

$\frac{\frac{-2x^2}{x^2} + \frac{7x}{x^2} - \frac{3}{x^2}}{\frac{x^2}{x^2} + \frac{2x}{x^2} - \frac{3}{x^2}} \to \lim_{x\to\infty} \frac{-2+\frac{7}{x}-\frac{3}{x^2}}{1+\frac{2}{x}-\frac{3}{x^2}}$

$= \frac{-2+0-0}{1+0-0} = -2$

OR H.A. = -2

2. $\int \frac{1}{x^2}\,dx =$

(A) $\ln x^2 + C$ (B) $-\ln x^2 + C$ (C) $x^{-1} + C$ (D) $-x^{-1} + C$ (E) $-2x^{-3} + C$

$\int x^{-2}\,dx$

$\frac{x^{-1}}{-1} + C$

$-\frac{1}{x} + C$

AP Calculus 2008 Multiple Choice

3. If $f(x) = (x-1)(x^2+2)^3$, then $f'(x) =$

Product Rule

(A) $6x(x^2+2)^2$

(B) $6x(x-1)(x^2+2)^2$

(C) $(x^2+2)^2(x^2+3x-1)$

(D) $(x^2+2)^2(7x^2-6x+2)$

(E) $-3(x-1)(x^2+2)^2$

$f'(x) = (1)(x^2+2)^3 + (x-1)(3(x^2+2)^2(2x))$

$= (x^2+2)^3 + 6x(x-1)(x^2+2)^2$

$= (x^2+2)^2[(x^2+2)+6x(x-1)]$

$= (x^2+2)^2[x^2+2+6x^2-6x]$

$= (x^2+2)^2[7x^2-6x+2]$

4. $\int(\sin(2x)+\cos(2x))\,dx =$

(A) $\frac{1}{2}\cos(2x)+\frac{1}{2}\sin(2x)+C$

(B) $-\frac{1}{2}\cos(2x)+\frac{1}{2}\sin(2x)+C$

(C) $2\cos(2x)+2\sin(2x)+C$

(D) $2\cos(2x)-2\sin(2x)+C$

(E) $-2\cos(2x)+2\sin(2x)+C$

$\frac{1}{2}\int 2\sin 2x\,dx + \frac{1}{2}\int 2\cos 2x\,dx$

$\frac{1}{2}[-\cos 2x] + \frac{1}{2}[\sin 2x]$

$-\frac{1}{2}\cos 2x + \frac{1}{2}\sin 2x + C$

5. $\lim_{x \to 0} \frac{5x^4+8x^2}{3x^4-16x^2}$ is

Factor $\frac{\cancel{x^2}(5x^2+8)}{\cancel{x^2}(3x^2-16)}$

(A) $-\frac{1}{2}$ (B) 0 (C) 1 (D) $\frac{5}{3}+1$ (E) nonexistent

$$\lim_{x \to 0} \frac{5x^2+8}{3x^2-16} = \frac{5(0)^2+8}{3(0)^2-16} = \frac{8}{-16} = -1/2$$

$$f(x)=\begin{cases} \frac{x^2-4}{x-2} & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases} \to \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

6. Let f be the function defined above. Which of the following statements about f are true?

✓ I. f has a limit at $x=2$.

X II. f is continuous at $x=2$.

X III. f is differentiable at $x=2$.

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

I) $\lim_{x \to 2^-} f(x) = 2+2 = 4$

$\lim_{x \to 2^+} f(x) = 2+2 = 4$

$\lim_{x \to 2} f(x) = 4$

II) f(x) is NOT cont. at x=2 b/c $f(2) = 1 \neq 4$

III) NOT Diff at x=2 b/c not cont. at x=2.

7. A particle moves along the x-axis with velocity given by $v(t)=3t^2+6t$ for time $t \geq 0$. If the particle is at position $x=2$ at time $t=0$, what is the position of the particle at $t=1$?

(A) 4 (B) 6 (C) 9 (D) 11 (E) 12

$$\int_0^1 v(t)\,dt = p(1) - p(0)$$

$$\int_0^1 3t^2+6t\,dt = p(1) - 2$$

$$\frac{\cancel{3}t^3}{\cancel{3}} + \frac{6t^2}{2}\Big]_0^1 = p(1) - 2$$

$$[(1)^3+3(1)^2] - [(0)^3+3(0)^2] = p(1) - 2$$

$4 = p(1) - 2$

$p(1) = 6$

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8. If $f(x)=\cos(3x)$, then $f'\left(\frac{\pi}{9}\right)=$

(A) $\frac{3\sqrt{3}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $-\frac{\sqrt{3}}{2}$ (D) $-\frac{3}{2}$ (E) $-\frac{3\sqrt{3}}{2}$

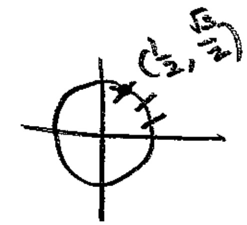
$f(x) = \cos(3x)$

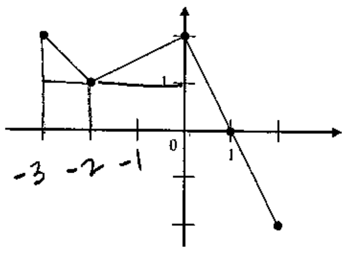
$f'(x) = -3\sin(3x)$

$f'(\pi/9) = -3\sin\left(3 \cdot \frac{\pi}{9}\right)$

$= -3\sin\left(\frac{\pi}{3}\right)$

$= -3\left(\frac{\sqrt{3}}{2}\right) = \frac{-3\sqrt{3}}{2}$





Graph of f

9. The graph of the piecewise linear function f is shown in the figure above. If $g(x)=\int_{-2}^{x} f(t)\,dt$, which of the following values is greatest?

(A) $g(-3)$ (B) $g(-2)$ (C) $g(0)$ (D) $g(1)$ (E) $g(2)$

$g(-3)=\int_{-2}^{-3} f(t)$

$=-\int_{-3}^{-2} f(t)$

$=-\left[\frac{1}{2}(1)(1)+(1)(1)\right]$

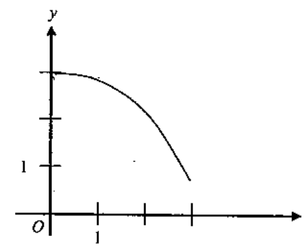
$=-1.5$

$g(-2)=\int_{-2}^{-2} f(t)=0$

$\int_{-2}^{0} f(t)=\frac{1}{2}(2)(1)+(2)(1)=3$

$\int_{-2}^{1} f(t)=3+\frac{1}{2}(1)(2)=4$

$\int_{-2}^{2} f(t)=4-\frac{1}{2}(2)(1)=3$



Graph of f

10. The graph of function f is shown above for $0 \le x \le 3$. Of the following, which has the least value?

(A) $\int_{1}^{3} f(x)\,dx$ actual area

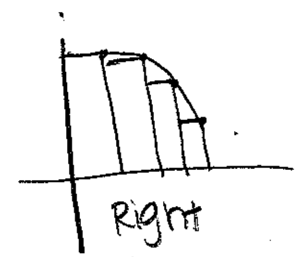
(B) Left Riemann sum approximation of $\int_{1}^{3} f(x)\,dx$ with 4 subintervals of equal length

(C) Right Riemann sum approximation of $\int_{1}^{3} f(x)\,dx$ with 4 subintervals of equal length

(D) Midpoint Riemann sum approximation of $\int_{1}^{3} f(x)\,dx$ with 4 subintervals of equal length

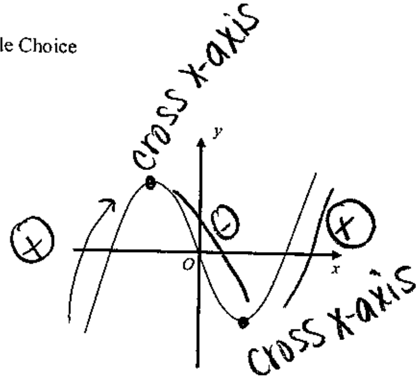
(E) Trapezoidal sum approximation of $\int_{1}^{3} f(x)\,dx$ with 4 subintervals of equal length

B) 
Left above

C) 
Right below

D) 
about equal

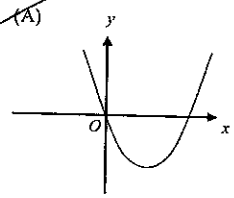
E) 



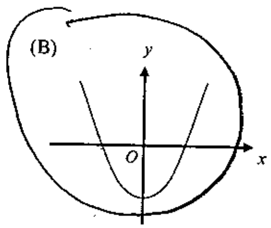
Graph of f

11. The graph of a function f is shown above. Which of the following could be the graph of f', the derivative of f?

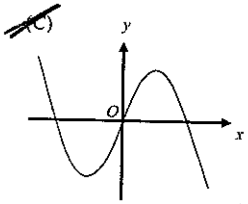
~~(A)~~



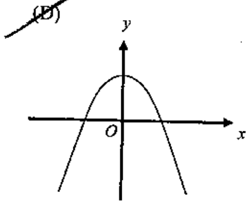
(B)



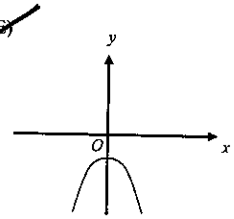
~~(C)~~



~~(D)~~



~~(E)~~



AP Calculus 2008 Multiple Choice

12. If $f(x) = e^{(2/x)}$, then $f'(x) =$

(A) $2e^{(2/x)} \ln x$ (B) $e^{(2/x)}$ (C) $e^{(-2/x^2)}$ (D) $-\frac{2}{x^2}e^{(2/x)}$ (E) $-2x^2e^{(2/x)}$

$f(x) = e^{2x^{-1}}$

$f'(x) = e^{2x^{-1}}(-2x^{-2}) = -\frac{2}{x^2}e^{\frac{2}{x}}$

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13. If $f(x) = x^2 + 2x$, then $\frac{d}{dx}(f(\ln x)) =$

(A) $\frac{2\ln x + 2}{x}$ (B) $2x\ln x + 2$ (C) $2\ln x + 2$ (D) $2\ln x + \frac{2}{x}$ (E) $\frac{2x+2}{x}$

$f(\ln x) = (\ln x)^2 + 2(\ln x)$

$\frac{d}{dx}(f(\ln x)) = 2(\ln x)'(\frac{1}{x}) + 2(\frac{1}{x})$

$= \frac{2}{x}\ln x + \frac{2}{x}$

$= \frac{2\ln x + 2}{x}$

x	0	1	2	3
$f''(x)$	5	0	−7	4

14. The polynomial function f has selected values of its second derivative f'' given in the table above. Which of the following statements must be true?

(A) f is increasing on the interval $(0, 2)$.

(B) f is decreasing on the interval $(0, 2)$.

(C) f has a local maximum at $x = 1$.

we don't know b/c we don't have f'(x).

(D) The graph of f has a point of inflection at $x = 1$. ← Since we don't know what is Right beside of 1, we don't know if it changes signs.

(E) The graph of f changes concavity in the interval $(0, 2)$.

$f''(x)$ goes from $\oplus \to \ominus$

15. $\int \frac{x}{x^2-4}\,dx =$

(A) $\frac{-1}{4\left(x^2-4\right)^2}+C$

(B) $\frac{1}{2\left(x^2-4\right)}+C$

(C) $\frac{1}{2}\ln\left|x^2-4\right|+C$

(D) $2\ln\left|x^2-4\right|+C$

(E) $\frac{1}{2}\arctan\left(\frac{x}{2}\right)+C$

$$\frac{1}{2}\int 2x\,(x^2-4)^{-1}\,dx \quad 2x$$

$$\frac{1}{2}\left[\frac{(x^2-4)^0}{0}\right]+C$$

$$\frac{1}{2}\ln|x^2-4|+C$$

Implicit Differentiation

16. If $\sin(xy) = x$, then $\frac{dy}{dx} =$

(A) $\frac{1}{\cos(xy)}$

(B) $\frac{1}{x\cos(xy)}$

(C) $\frac{1-\cos(xy)}{\cos(xy)}$

(D) $\frac{1-y\cos(xy)}{x\cos(xy)}$

(E) $\frac{y(1-\cos(xy))}{x}$

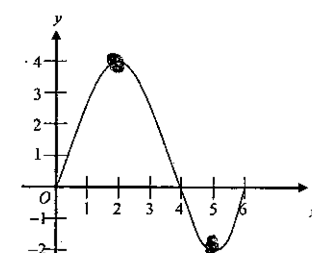
$$\sin(xy) = x$$

$$\cos(xy)\cdot\left[(1)(y)+(x)(1\tfrac{dy}{dx})\right] = 1$$

$$y\cos(xy) + x\frac{dy}{dx}\cos(xy) = 1$$

$$x\frac{dy}{dx}\cos(xy) = 1-y\cos(xy)$$

$$\frac{dy}{dx} = \frac{1-y\cos(xy)}{x\cos(xy)}$$



Graph of f

17. The graph of the function f shown above has horizontal tangents at $x = 2$ and $x = 5$. Let g be the function defined by $g(x) = \int_0^x f(t)\,dt$. For what values of x does the graph of g have a point of inflection?

(A) 2 only (B) 4 only (C) 2 and 5 only (D) 2, 4, and 5 (E) 0, 4, and 6

$g'(x) = f(x)(1)$ ← and Fund. Thm of Calc

$g''(x) = f'(x)$

P.O.I when $g''(x)$ or $f'(x)$ changes signs → rel. max/min on f(x)

★work backwards★

$y = \ominus x + k$ → slope = -1

18. In the xy-plane, the line $x + y = k$, where k is a constant, is tangent to the graph of $y = x^2 + 3x + 1$. What is the value of k?

(A) −3 (B) −2 (C) −1 (D) 0 (E) 1

$f(x) = x^2 + 3x + 1$

$f'(x) = 2x + 3$

$2x + 3 = -1$ ← slope

$2x = -4$

$x = -2$ → point $(-2, -1)$

$(-2)^2 + 3(-2) + 1$

$y + 1 = -1(x + 2)$

$y + 1 = -x - 2$

$\boxed{y = -x - 3}$

★★ 19. What are all horizontal asymptotes of the graph of $y = \frac{5+2^x}{1-2^x}$ in the xy-plane?

(A) $y = -1$ only

(B) $y = 0$ only

(C) $y = 5$ only

(D) $y = -1$ and $y = 0$

(E) $y = -1$ and $y = 5$

$\lim_{x \to \infty} \frac{5 + 2^x}{1 - 2^x}$

divide each term by 2^x

$\frac{\frac{5}{2^x} + \frac{2^x}{2^x}}{\frac{1}{2^x} - \frac{2^x}{2^x}} \rightarrow \lim_{x \to \infty} \frac{\frac{5}{2^x} + 1}{\frac{1}{2^x} - 1} = \frac{0 + 1}{0 - 1} = \frac{1}{-1} = -1$

Right end behavior

Left End Behavior

$\lim_{x \to -\infty} \frac{5 + 2^x}{1 - 2^x}$

or $\lim_{x \to \infty} \frac{5 + \frac{1}{2^x}}{1 - \frac{1}{2^x}} = \frac{5 + 0}{1 - 0} = 5$

20. Let f be a function with a second derivative given by $f''(x) = x^2(x-3)(x-6)$. What are the x-coordinates of the points of inflection of the graph of f?

(A) 0 only

(B) 3 only

(C) 0 and 6 only

(D) 3 and 6 only

(E) 0, 3, and 6

$f''(x) = 0$ & changes signs

$f''(x) = x^2(x-3)(x-6)$

$x = 0$ $x = 3$ $x = 6$

-1, 2, 4, 7

(+)(−)(−) pos | 0 | (+)(−)(−) pos | 3 | (+)(+)(−) neg | 6 | (+)(+)(+) pos

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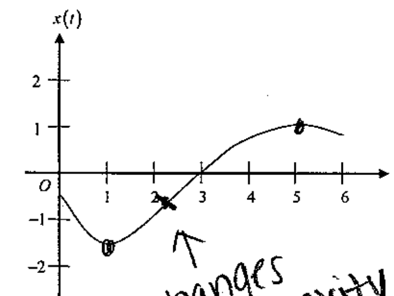


f(x)

X(t)

changes concavity

P.O.I at $t = 2$

21. A particle moves along a straight line. The graph of the particle's position $x(t)$ at time t is shown above for $0 < t < 6$. The graph has horizontal tangents at $t = 1$ and $t = 5$ and a point of inflection at $t = 2$. For what values of t is the velocity of the particle increasing?

(A) $0 < t < 2$

(B) $1 < t < 5$

(C) $2 < t < 6$

(D) $3 < t < 5$ only

(E) $1 < t < 2$ and $5 < t < 6$

$f'(x)$ = velocity

$f'(x)$ is incr when $f''(x)$ is positive

→ $f(x)$ is concave up.

TOTAL PPl

mult.

22. A rumor spreads among a population of N people at a rate proportional to the product of the number of people who have heard the rumor and the number of people who have not heard the rumor. If p denotes the number of people who have heard the rumor, which of the following differential equations could be used to model this situation with respect to time t, where k is a positive constant?

(A) $\frac{dp}{dt} = kp$

(B) $\frac{dp}{dt} = kp(N-p)$

(C) $\frac{dp}{dt} = kp(p-N)$

(D) $\frac{dp}{dt} = kt(N-t)$

(E) $\frac{dp}{dt} = kt(t-N)$

k[(# ppl who heard)(# who haven't)]

$$\frac{dp}{dt} = k[(p)(N-p)]$$

$$\frac{dp}{dt} = kp(N-p)$$

↑

k is a constant

AP Calculus 2008 Multiple Choice

23. Which of the following is the solution to the differential equation $\frac{dy}{dx} = \frac{x^2}{y}$ with the initial condition $y(3) = -2$?

(A) $y = 2e^{-9+x^3/3}$

(B) $y = -2e^{-9+x^3/3}$

(C) $y = \sqrt{\frac{2x^3}{3}}$

(D) $y = \sqrt{\frac{2x^3}{3} - 14}$

(E) $y = -\sqrt{\frac{2x^3}{3} - 14}$

$$\int y\,dy = \int x^2\,dx$$

$$\frac{1}{2}y^2 = \frac{1}{3}x^3 + C$$

$$\frac{1}{2}(-2)^2 = \frac{1}{3}(3)^3 + C$$

$$2 = 9 + C$$

$$C = -7$$

$$\frac{1}{2}y^2 = \frac{1}{3}x^3 - 7$$

$$y^2 = \frac{2}{3}x^3 - 14$$

$$y = -\sqrt{\frac{2}{3}x^3 - 14}$$

24. The function f is twice differentiable with $f(2) = 1$, $f'(2) = 4$, and $f''(2) = 3$. What is the value of the approximation of $f(1.9)$ using the line tangent to the graph of f at $x = 2$?

(A) 0.4 (B) 0.6 (C) 0.7 (D) 1.3 (E) 1.4

$f(2) = 1 \rightarrow$ point $(2, 1)$

$f'(2) = 4 \rightarrow$ slope of tangent $= 4$

$y - 1 = 4(x - 2)$

plug in 1.9 for x

$y - 1 = 4(1.9 - 2)$

$y - 1 = 4(-.1)$

$y - 1 = -.4$

$y = .6$

$$f'(x) = \begin{cases} c, & x \le 2 \\ 2x - c, & x > 2 \end{cases}$$

$$f(x) = \begin{cases} cx + d & \text{for } x \le 2 \\ x^2 - cx & \text{for } x > 2 \end{cases}$$

25. Let f be the function defined above, where c and d are constants. If f is differentiable at $x = 2$, what is the value of $c + d$?

(A) −4 (B) −2 (C) 0 (D) 2 (E) 4

① f(2) defined ✓

② For f(x) to be continuous at x = 2,

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$

$c(2) + d = (2)^2 - c(2)$

$2c + d = 4 - 2c$

③ For f(x) to be diff at x = 2 →

$\lim_{x\to 2^-} f'(x) = \lim_{x\to 2^+} f'(x)$

$c = 2(2) - c$

$c = 4 - c$

$2c = 4$

$\boxed{c = 2}$

$2c + d = 4 - 2c$

$2(2) + d = 4 - 2(2)$

$4 + d = 0$

$\boxed{d = -4}$

$c + d = 2 + -4$

$c + d = -2$

** 26. What is the slope of the line tangent to the curve $y = \arctan(4x)$ at the point at which $x = \frac{1}{4}$?

(A) 2 (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{1}{2}$ (E) −2

$y = \tan^{-1}(4x)$

$\frac{dy}{dx} = \frac{1}{1+(4x)^2}(4) = \frac{4}{1+16x^2}$

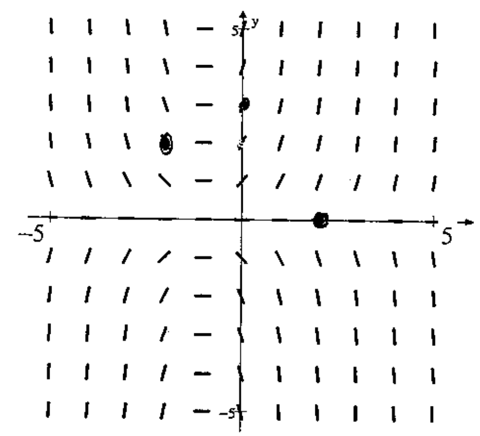
plug in 1/4

slope $= \frac{4}{1+16(\frac{1}{4})^2}$

$= \frac{4}{1+16(\frac{1}{16})}$

$= \frac{4}{2} \rightarrow$ slope $= 2$

$$\boxed{\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}}$$



27. Shown above is a slope field for which of the following differential equations?

~~(A) $\frac{dy}{dx} = xy$~~

~~(B) $\frac{dy}{dx} = xy - y$~~

(C) $\frac{dy}{dx} = xy + y$

~~(D) $\frac{dy}{dx} = xy + x$~~

~~(E) $\frac{dy}{dx} = (x+1)^3$~~

(2, 0) slope = 0

a) (2)(0) = 0 ✓

b) (2)(0) − 0 = 0 ✓

c) (2)(0) + 0 = 0 ✓

~~d) (2)(0) + 2 = 2~~ x

~~e) $(2+1)^3 = 9$~~ x

(0, 3) → slope (+)

~~ⓐ (0)(3) = 0~~

~~ⓑ (0)(3) − 3 = −3~~

ⓒ (0)(3) + 3 = 3 ✓

(−2, 2) → slope (−)

(−2)(2) + 2 = −2 ✓

inverse
switch x & y

$g(15)=3$ $g(3)=6$

28. Let f be a differentiable function such that $f(3)=15$, $f(6)=3$, $f'(3)=-8$, and $f'(6)=-2$. The function g is differentiable and $g(x)=f^{-1}(x)$ for all x. What is the value of $g'(3)$?

$g(x)$ = inverse of $f(x)$

(A) $-\frac{1}{2}$

(B) $-\frac{1}{8}$

(C) $\frac{1}{6}$

(D) $\frac{1}{3}$

(E) The value of $g'(3)$ cannot be determined from the information given.

$$\frac{d}{dx}f^{-1}(x) = \frac{1}{f'(f^{-1}(x))} \rightarrow g'(x) = \frac{1}{f'(g(x))}$$

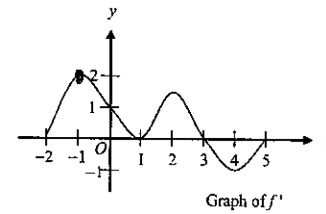
$$g'(3) = \frac{1}{f'(g(3))} = \frac{1}{f'(6)} = \frac{1}{-2}$$

Remember:

$$\boxed{\frac{d}{dx}f^{-1}(x) = \frac{1}{f'(f^{-1}(x))}}$$

* Inverse Derivative *

Section 2: Calculator (#'s 76-92)



Graph of f'

$f'(x)$

76. The graph of f', the derivative f, is shown above for $-2 \le x \le 5$. On what intervals is f increasing?

(A) $[-2, 1]$ only

(B) $[-2, 3]$

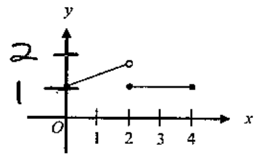
(C) $[3, 5]$ only

(D) $[0, 1.5]$ and $[3, 5]$

(E) $[-2, -1]$, $[1, 2]$, and $[4, 5]$

$f(x)$ is increasing when $f'(x) > 0$

$(-2, 1) \cup (1, 3)$



Graph of f

77. The figure above shows the graph of a function f with domain $0 \le x \le 4$. Which of the following statements are true?

I. $\lim_{x \to 2^-} f(x)$ exists. $= 2$

II. $\lim_{x \to 2^+} f(x)$ exists. $= 1$

III. $\lim_{x \to 2} f(x)$ exists. $\rightarrow$ Not True b/c $\lim_{x \to 2^-} f(x) \neq \lim_{x \to 2^+} f(x)$

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

78. The first derivative of the function f is defined by $f'(x) = \sin(x^3 - x)$ for $0 \le x \le 2$. On what interval(s) is f increasing?

(A) $1 \le x \le 1.445$

(B) $1 \le x \le 1.691$

(C) $1.445 \le x \le 1.875$

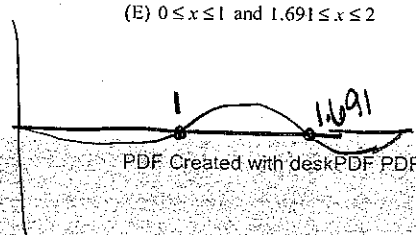
(D) $0.577 \le x \le 1.445$ and $1.875 \le x \le 2$

(E) $0 \le x \le 1$ and $1.691 \le x \le 2$

Graph on calc.

f(x) is increasing when f(x) > 0 [above x-axis]

(1, 1.691)



FLIP

79. If $\int_{-5}^{2} f(x)\,dx = -17$ and $\int_{5}^{2} f(x)\,dx = -4$, what is the value of $\boxed{\int_{-5}^{5} f(x)\,dx}$?

(A) -21 (B) -13 (C) 0 (D) 13 (E) 21

$\int_{-5}^{2} f(x) = -17$ $\quad \underbrace{\int_{2}^{5} f(x) = 4}_{\text{switch a \& b} \rightarrow \text{take out negative}}$

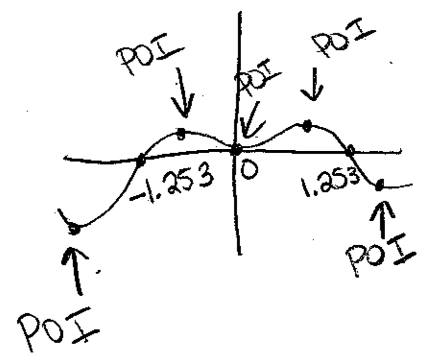
$\int_{-5}^{5} f(x)dx = \int_{-5}^{2} f(x) + \int_{2}^{5} f(x)$

$= -17 + 4 = -13$

80. The derivative of the function f is given by $f'(x) = x^2 \cos(x^2)$. How many points of inflection does the graph of f have on the open interval $(-2, 2)$?

Graph on CALC

(A) One (B) Two (C) Three (D) Four (E) Five



f(x) has inflection points when f''(x) = 0 & changes signs →

f'(x) goes from incr → decr or decr → incr (rel. max / min)

81. If $G(x)$ is an antiderivative for $f(x)$ and $G(2)=-7$, then $G(4)=$

(A) $f'(4)$

(B) $-7+f'(4)$

(C) $\int_2^4 f(t)\,dt$

(D) $\int_2^4 (-7+f(t))\,dt$

(E) $-7+\int_2^4 f(t)\,dt$

$$\int_2^4 f(x) = G(4)-G(2)$$

$$\int_2^4 f(x) = G(4)-(-7)$$

$$G(4) = -7+\int_2^4 f(x)\,dx$$

---

82. A particle moves along a straight line with velocity given by $v(t)=7-(1.01)^{-t^2}$ at time $t\ge 0$. What is the acceleration of the particle at time $t=3$?

velocity

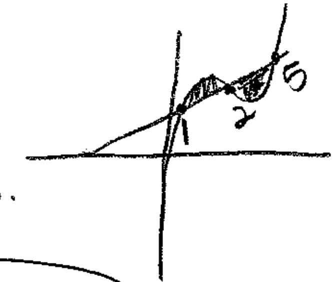
(A) -0.914 (B) 0.055 (C) 5.486 (D) 6.086 (E) 18.087

Find the derivative.

$v(t) = 7-(1.01)^{-t^2}$ ← use $\boxed{\text{math } 8}$ on CALC

$a(t) = v'(t)$

$v'(3) = 0.055$



AP Calculus 2008 Multiple Choice

integral Graph on calc.

83. What is the area enclosed by the curves $y=x^3-8x^2+18x-5$ and $y=x+5$?

use math 9

(A) 10.667

(B) 11.833

(C) 14.583

(D) 21.333

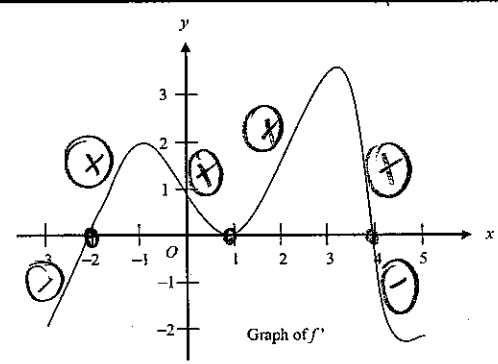
(E) 32

$$\int_1^2 [x^3-8x^2+18x-5]-[x+5]\,dx = .583$$

$$+$$

$$\int_2^5 [x+5]-[x^3-8x^2+18x-5]\,dx = 11.25$$

$$.583+11.25 = 11.833$$



Graph of f'

$f'(x)$

84. The graph of the derivative of a function f is shown in the figure above. The graph has horizontal tangent lines at $x=-1$, $x=1$, and $x=3$. At which of the following values of x does f have a relative maximum?

(A) -2 only

(B) 1 only

(C) 4 only

(D) -1 and 3 only

(E) -2, 1, and 4

f(x) has a rel. max when f'(x) changes from + to −

x	-4	-3	-2	-1
$f(x)$	0.75	-1.5	-2.25	-1.5
$f'(x)$	-3	-1.5	0	1.5

85. The table above gives values of a function f and its derivative at selected values of x. If f' is continuous on the interval $[-4, -1]$, what is the value of $\int_{-4}^{-1} f'(x)\,dx$?

(A) -4.5 (B) -2.25 (C) 0 (D) 2.25 (E) 4.5

$$\int_{-4}^{-1} f'(x)dx = f(-1) - f(-4)$$

$$= -1.5 - (0.75)$$

$$= +2.25$$

v(t)

rel. max at x=3

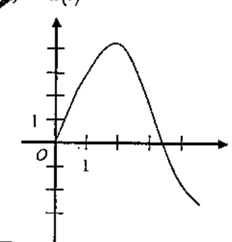
t	0	1	2	3	4
$v(t)$	-1	2	3	0	-4

decr incr (+) decr (−)

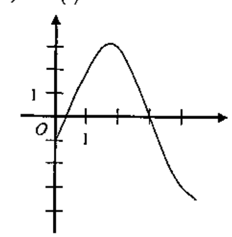
86. The table gives selected values of the velocity, $v(t)$, of a particle moving along the x-axis. At time $t = 0$, the particle is at the origin. Which of the following could be the graph of the position, $x(t)$, of the particle for $0 \le t \le 4$?

position graphs

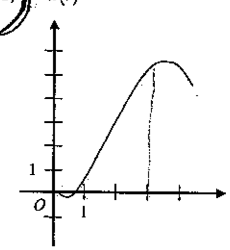
(A) $x(t)$



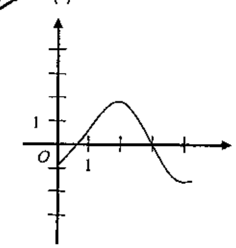
(B) $x(t)$



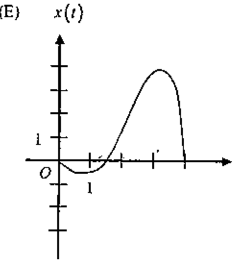
(C) $x(t)$



(D) $x(t)$



(E) $x(t)$



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87. An object traveling in a straight line has position $x(t)$ at time t. If the initial position is $x(0)=2$ and the velocity of the object is $v(t)=\sqrt[3]{1+t^2}$, what is the position of the object at time $t=3$?

(A) 0.431 (B) 2.154 (C) 4.512 (D) 6.512 (E) 17.408

$\int_0^3 v(t)dt = P(3) - P(0)$

use math 9 → $\underbrace{\int_0^3 \sqrt[3]{1+t^2}\,dt}$ $= P(3) - 2$

$4.512 = P(3) - 2 \qquad P(3) = 6.512$

88. The radius of a sphere is decreasing at a rate of 2 centimeters per second. At the instant when the radius of the sphere is 3 centimeters, what is the rate of change, in square centimeters per second, of the surface area of the sphere? (The surface area S of a sphere with radius r is $S=4\pi r^2$)

(A) -108π (B) -72π (C) -48π (D) -24π (E) -16π

$S = 4\pi r^2$

$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$

$\frac{dr}{dt} = -2$ cm/sec

$r = 3$

$\frac{dS}{dt} = 8\pi(3)(-2)$

$= -48\pi$

Rolle's Thm
- continuous ✓
- differentiable

89. The function f is continuous ✓ for $-2 \le x \le 2$ and $f(-2)=f(2)=0$. If there is no c, where $-2<c<2$, for which $f'(c)=0$, which of the following statements must be true?

(A) For $-2<k<2$, $f'(k)>0$.

(B) For $-2<k<2$, $f'(k)<0$.

(C) For $-2<k<2$, $f'(k)$ exists.

(D) For $-2<k<2$, $f'(k)$ exists, but f' is not continuous.

(E) For some k, where $-2<k<2$, $f'(k)$ does not exist. not differentiable

90. The function f is continuous on the closed interval [2, 4] and twice differentiable on the open interval (2, 4). If $f'(3)=2$ and $f''(x)<0$ on the open interval (2,4), which of the following could be a table of values for f?

(A)

x	$f(x)$
2	2.5
3	5
4	6.5

2.5, 1.5

$f'(3) = \frac{6.5-2.5}{4-2} = 2$ ✓

(B)

x	$f(x)$
2	2.5
3	5
4	7

2.5, 2

(C)

x	$f(x)$
2	3
3	5
4	6.5

2, 1.5

~~(D)~~

x	$f(x)$
2	3
3	5
4	7

2, 2

~~(E)~~

x	$f(x)$
2	3.5
3	5
4	7.5

1.5, 2.5 → f(x) incr

$f'(3) = 2$ means f(x) is incr.

$f''(x) < 0$ means f(x) is concave down

↓

means $f'(x)$ is decr

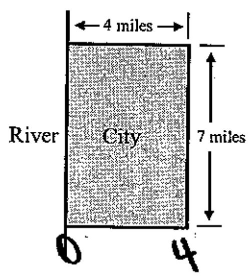
91. What is the average value of $y = \dfrac{\cos x}{x^2+x+2}$ on the closed interval $[-1, 3]$?

(A) -0.085 (B) 0.090 (C) 0.183 (D) 0.244 (E) 0.732

Avg. Value:

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

$$\frac{1}{3-(-1)}\int_{-1}^{3} \frac{\cos x}{x^2+x+2}\,dx = \frac{1}{4}[0.732] = 0.183$$



92. A city located beside a river has a rectangular boundary as shown in the figure above. The population density of the city at any point along a strip x miles from the river's edge is $f(x)$ persons per square mile. Which of the following expressions gives the population of the city?

(A) $\int_0^4 f(x)\,dx$

(B) $7\int_0^4 f(x)\,dx$

(C) $28\int_0^4 f(x)\,dx$

(D) $\int_0^7 f(x)\,dx$

(E) $4\int_0^7 f(x)\,dx$

persons per sq mile

$$\text{Area} = \int_0^4 (7 f(x))\,dx$$

$$= 7\int_0^4 f(x)\,dx$$